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The relevance of the axial current for pion production processes off the nucleon with real or virtual photons is revisited. Employing the hypothesis of a partially conserved axial current (PCAC), it is shown that, when all of the relevant contributions are taken into account, PCAC does not provide any additional constraint for threshold production processes that goes beyond the Goldberger–Treiman relation. In particular, it is shown that pion electroproduction processes at threshold cannot be used to extract any information regarding the weak axial form factor. The relationships found in previous investigations are seen to be an accident of the approximations usually made in this context.

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The hypothesis of a partially conserved axial current (PCAC) [1] has been employed in many investigations for constraining scattering processes involving pions at threshold. One of its early successes was the relation by Goldberger and Treiman [2] between the strength of the weak decay of the nucleon g_A and the strong-interaction πNN coupling constant $g_{\pi NN}$, i.e.,

$$\frac{g_A}{f_\pi} \approx \frac{g_{\pi NN}}{m}, \quad (1)$$

where f_π is the weak decay constant of the pion and m is the nucleon mass. Experimentally this relation is found to be satisfied to better than 10%.

Recently, PCAC relations were employed to extract the properties of the nucleon’s weak decay axial form factor G_A from threshold pion electroproduction data (see [3,4], and references therein). These extractions are based on the assumption that G_A is related to the electromagnetic structure of the Kroll–Ruderman contact term [5–11].

I will show here that the previous derivations of this relationship are based on an incomplete evaluation of the relevant PCAC expressions and that, if all mechanisms are taken into account, the dependence on G_A vanishes. From the present derivation, it will become obvious that the identification of G_A as the form factor entering the Kroll–Ruderman term is an accident of the usually employed approximations.

To set the stage, I will briefly recapitulate the basic PCAC relations [1]. Excluding ‘second class’ (i.e., tensor) currents, the general form of the weak axial current is given by

$$j_A^\mu = -\bar{u}_f \gamma_5 \left[\gamma^\mu G_A + (p - p')^\mu G_P \right] u_i \frac{\tau}{2}. \quad (2)$$

The PCAC hypothesis constrains this current by

$$(p' - p)_\mu j_A^\mu = -\frac{f_\pi \mu^2}{t - \mu^2} \bar{u}_f \gamma_5 G_t u_i \tau, \quad (3)$$

which provides a conserved current for vanishing pion mass μ . G_t is the πNN vertex function; other than the γ_5 which has been pulled out explicitly, I make no assumptions about the internal structure of G_t . Of course, within the present context, i.e., between on-shell spinors, G_t is a function of $t = (p - p')^2$ only, where p and p' are the initial and final nucleon momenta, respectively. τ is the vertex isospin operator. Here and throughout the present work, I employ a Cartesian isospin basis, and I suppress the corresponding indices; summation over these indices is implied when quantities carrying isospin indices are multiplied with each other.

The two form factors of the axial current are related via Eq. (3), i.e.,

$$2mG_A + tG_P = -2f_\pi \frac{\mu^2}{t - \mu^2} G_t. \quad (4)$$

Evaluated at $t = 0$, this provides the Goldberger–Treiman relation [2],

$$g_0 \equiv G_t(0) = \frac{m}{f_\pi} G_A(0), \quad (5)$$

where I have defined the constant g_0 to be used below. Equation (1) assumes that $G_A(0) \approx g_A$ and $G_t(0) \approx g_{\pi NN}$ since the pion mass is small.

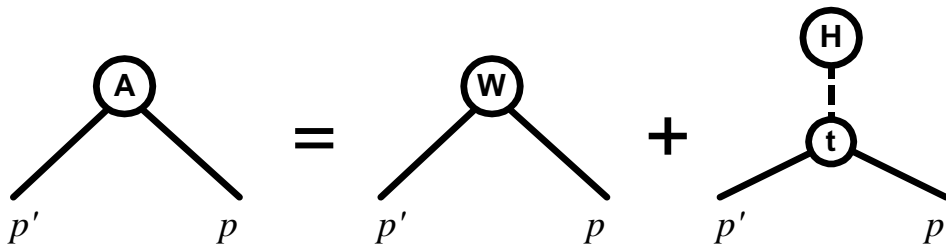


FIG. 1. Splitting of the axial current \hat{j}_A into a conserved weak part $\hat{j}_{A,W}$ and a pion-pole-dominated hadronic part $\hat{j}_{A,H}$; the latter produces the PCAC divergence of Eq. (3). Here, and in all other diagrams, time proceeds from right to left.

Note that this derivation presumes that G_p is less singular than $1/t$. In view of the reasonable experimental verification of the Goldberger–Treiman relation, this therefore means that

$$G_A \approx -\frac{f_\pi}{m} \frac{\mu^2}{t - \mu^2} G_t, \quad \text{for small } t, \quad (6)$$

should be an acceptable generalization of the Goldberger–Treiman relation.

While strictly speaking, Eq. (3) is presumed to be valid only for t values up to order μ^2 [1], I will in the following take all of the preceding relations at face value, assuming them to be valid at the operator level, and—if necessary at all—will look at limits of small t , etc., only at the end.

Introducing an operator \hat{j}_A^μ for the axial current, i.e.,

$$\hat{j}_A^\mu = \bar{u}_f \hat{j}_A^\mu u_i, \quad (7)$$

it can be split into weak and hadronic parts according to

$$\hat{j}_A^\mu = \hat{j}_{A,W}^\mu + \hat{j}_{A,H}^\mu, \quad (8)$$

where

$$\hat{j}_{A,W}^\mu = -g_0 f_\pi \gamma_5 \left[\frac{\gamma^\mu}{2m} + \frac{(p' - p)^\mu}{t} \right] \tilde{G}_{A\tau}, \quad (9a)$$

$$\hat{j}_{A,H}^\mu = -g_0 f_\pi \gamma_5 \frac{(p' - p)^\mu}{t} \frac{\mu^2}{t - \mu^2} \tilde{G}_{t\tau}; \quad (9b)$$

G_p was eliminated using Eq. (4). The tilde symbolizes form factors $\tilde{G}_A = G_A(t)/G_A(0)$ and $\tilde{G}_t = G_t(t)/G_t(0)$ which are normalized to unity at $t = 0$.

Although individually the weak and the hadronic parts of the current each contain a singularity at $t = 0$, their sum obviously does not. The divergence of the weak part,

$$(p' - p)_\mu \hat{j}_{A,W}^\mu = g_0 f_\pi \frac{\gamma_5 (\not{p} - m) + (\not{p}' - m) \gamma_5}{2m} \tilde{G}_{A\tau}, \quad (10)$$

which vanishes between nucleon spinors, yields the conserved part of the current and

$$(p' - p)_\mu \hat{j}_{A,H}^\mu = -g_0 f_\pi \frac{\gamma_5 \mu^2}{t - \mu^2} \tilde{G}_{t\tau} \quad (11)$$

provides the PCAC divergence of Eq. (3).

Note that the two contributions $\hat{j}_{A,W}^\mu$ and $\hat{j}_{A,H}^\mu$ may be interpreted as resulting from the two diagrams of Fig. 1. The hadronic current $\hat{j}_{A,H}^\mu$, in particular, provides the straightforward interpretation of the pion-pole-dominated diagram of Fig. 1: It describes the creation of the pion of mass μ out of the vacuum, with coupling operator $-f_\pi(p' - p)^\mu$ and associated normalized ‘form factor’ μ^2/t , and the subsequent propagation of the pion and its final absorption in the nucleon. In other words,

$$\hat{j}_\pi^\mu = -f_\pi \hat{q}^\mu \frac{\mu^2}{\hat{q}^2} \quad (12)$$

corresponds to the circle label H in Fig. 1, with $\hat{q} = p' - p$ being the pion's four-momentum flowing out of H.

The preceding equations provide all elements necessary for employing the axial current at the operator level suitable for a description in terms of Feynman diagrams.

I turn now to the main issue of the present work, the production of pions off the nucleon with real or virtual photons. The corresponding amplitude \mathcal{M} is determined by the four diagrams in Fig. 2, i.e.,

$$\mathcal{M} = \bar{u}_f (M_s^\nu + M_u^\nu + M_t^\nu + M_{\text{int}}^\nu) u_i \varepsilon_\nu. \quad (13)$$

Adapting the PCAC hypothesis to this process, one finds that \mathcal{M} satisfies [7,9,11]

$$\frac{f_\pi \mu^2}{q^2 - \mu^2} \mathcal{M} = q_\mu J_{A,\gamma}^{\mu\nu} \varepsilon_\nu - Q_\pi j_A^\nu \varepsilon_\nu, \quad (14)$$

where $J_{A,\gamma}^{\mu\nu}$ describes the coupling of the photon to the axial current and j_A^ν is the nucleon matrix element (7) of the axial current. $(Q_\pi)_{kl} = ei\varepsilon_{k3l}$ is the pion charge operator. Note that only the nucleons are on-shell here, but the pion is off-shell.

This relation between the pion photoproduction amplitude \mathcal{M} and the axial current is presumed to be valid only in the limit of vanishing pion momentum q . In the soft-pion limit $q \rightarrow 0$, following Ref. [8], the first term on the right-hand side here is often taken as zero by first considering the initial and final nucleon masses to be different, then letting q go to zero and then letting the mass difference go to zero.

Before I proceed to show that this limiting procedure is incorrect, let me first consider the second term, i.e.,

$$Q_\pi j_A^\nu \varepsilon_\nu = -g_0 f_\pi \bar{u}_f \gamma_5 \left[\frac{\gamma^\nu}{2m} \tilde{G}_A + \frac{(p' - p)^\nu}{t - \mu^2} \tilde{G}_t + (p' - p)^\nu \frac{\tilde{G}_A - \tilde{G}_t}{t} \right] u_i \varepsilon_\nu e_\pi, \quad (15)$$

where $e_\pi = Q_\pi \tau$ effectively describes the charge of the (outgoing) pion in a Cartesian basis.

Using $(p' - p) \cdot \varepsilon \rightarrow k \cdot \varepsilon = 0$, the result of taking the limit $q \rightarrow 0$,

$$Q_\pi j_A^\nu \varepsilon_\nu|_{q=0} = -g_0 f_\pi \bar{u}_f \gamma_5 \frac{\gamma^\nu}{2m} \tilde{G}_A(k^2) u_i \varepsilon_\nu e_\pi, \quad (16)$$

looks exactly like the Kroll-Ruderman contact current for pion photoproduction [5]. Since one indeed would have

$$f_\pi \mathcal{M}|_{q=0} = Q_\pi j_A^\nu \varepsilon_\nu|_{q=0}$$

if the first term could be dropped, this is used as the starting point when trying to extract the threshold behavior of pion production processes from $Q_\pi j_A^\nu \varepsilon_\nu$ by considering expansions around $q = 0$. In particular, one takes this as evidence that the electromagnetic structure of the Kroll-Ruderman term is described by the axial form factor \tilde{G}_A since it multiplies the $\gamma_5 \gamma^\nu$ operator [8–11].

To prove that these arguments are incorrect, I will actually explicitly derive Eq. (14) in a way which shows that this relation is devoid of any additional dynamical content that is not already part of the original pion-production amplitude. To this end I will consider the divergence of the current $J_{A,\gamma}^{\mu\nu}$ of Eq. (14). Instead of evaluating this in the usual manner by the LSZ reduction scheme [1,7,9], it is much more convenient to do this in terms of Feynman diagrams, consistent with the operator approach adopted here for the axial currents.

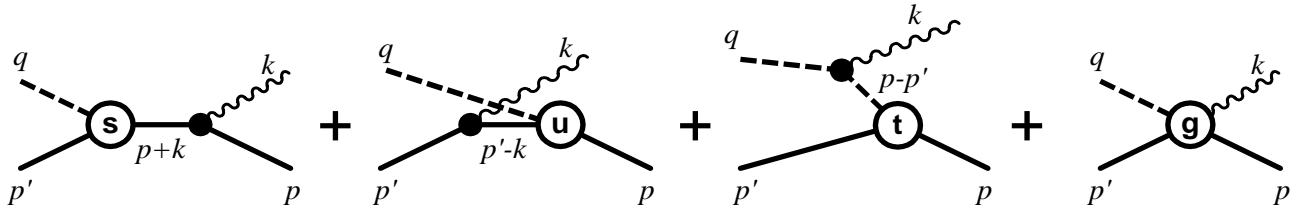


FIG. 2. Pion photoproduction for real or virtual photons. The last diagram marked g depicts the interaction current M_{int}^μ ; it subsumes the Kroll-Ruderman contact term, exchange-current contributions, and final-state interactions. The sum of all four diagrams is gauge-invariant [12,13].

Using the gauge-derivative method of Ref. [12], the current $J_{A,\gamma}^{\mu\nu}$ corresponds to inserting photon lines in all possible places in the axial-current diagrams of Fig. 1. The result is shown in Fig. 3, and one then easily reads off that

$$\begin{aligned}\hat{J}_{A,\gamma}^{\mu\nu} = & \hat{J}_{A,f}^{\mu} \frac{1}{\not{p} + \not{k} - m} \Gamma_i^{\nu} + \Gamma_f^{\nu} \frac{1}{\not{p}' - \not{k} - m} \hat{J}_{A,i}^{\mu} \\ & + \hat{J}_{\pi}^{\mu} \frac{1}{q^2 - \mu^2} \Gamma_{\pi}^{\nu} \frac{1}{t - \mu^2} \gamma_5 G_t \tau + W^{\mu\nu} \\ & + H^{\mu\nu} \frac{1}{t - \mu^2} \gamma_5 G_t \tau + \hat{J}_{\pi}^{\mu} \frac{1}{q^2 - \mu^2} M_{\text{int}}^{\nu}\end{aligned}\quad (17)$$

provides an operator expression for these diagrams. The operators $W^{\mu\nu}$ and $H^{\mu\nu}$ describe the respective contact terms from the second line of Fig. 3. $H^{\mu\nu}$ is given by

$$\begin{aligned}H^{\mu\nu} & \equiv -\{\hat{J}_{\pi}^{\mu}(p' - p)\}^{\nu} \\ & = -f_{\pi}\mu^2 \left[g^{\mu\nu} - \frac{q^{\mu}(2q - k)^{\nu}}{q^2} \right] \frac{Q_{\pi}}{t},\end{aligned}\quad (18)$$

where $-\{\hat{J}_{\pi}^{\mu}\}^{\nu}$ is the gauge-derivative notation of Ref. [12] which describes the coupling of the photon to \hat{J}_{π}^{μ} of Eq. (12). For $W^{\mu\nu}$, defined analogously as

$$W^{\mu\nu} \equiv -\{\hat{J}_{A,w}^{\mu}(p' - p)\}^{\nu}, \quad (19)$$

one cannot give a result in closed form since the internal structure of G_A is unknown.

In Eq. (17), Γ_{π}^{ν} is the electromagnetic current for the pion and the current for the nucleon is

$$\Gamma_N^{\nu} = \gamma^{\nu} Q_N + T_N^{\nu}, \quad (20a)$$

$$T_N^{\nu} = (\gamma^{\nu} k^2 - k^{\nu} \not{k}) \frac{F_1 - 1}{k^2} Q_N + i \frac{\sigma^{\nu\lambda} k_{\lambda}}{2m} \hat{\kappa}_N F_2. \quad (20b)$$

The form of the transverse current T_N^{ν} here is mandated by gauge-invariance requirements [12,13] (using the usual current without the k^{ν} term leads to incomplete cancellations and an incorrect threshold behavior, as in [7]). $N = i, f$ denotes the initial or the final nucleon; F_1 and F_2 are the usual Dirac and Pauli form factors; Q_N and $\hat{\kappa}_N$ are the nucleon charge and anomalous magnetic moment operators. Note that $\tau Q_i = e_i$ and $Q_f \tau = e_f$ provide effective (Cartesian-basis) charge operators for the nucleons in the present context and that one has $e_i = e_f + e_{\pi}$, describing charge conservation across the πNN vertex.

Of particular importance in Eq. (17) is the interaction current M_{int}^{μ} which originates from the photon attaching itself within the t -channel πNN vertex of the pion-pole-dominated diagram of Fig. 1. In lowest order (bare vertices), this corresponds to the usual gauge-invariance-preserving Kroll–Ruderman term as obtained by minimal substitution. In higher orders, with fully dressed vertices, this term contains a dressed Kroll–Ruderman term [cf. Eq. (26) below], exchange currents, and all contributions from final-state interactions [12,13].

In evaluating the divergence

$$(p' - p - k)_{\mu} \bar{u}_f \hat{J}_{A,\gamma}^{\mu\nu} u_i \varepsilon_{\nu} = -q_{\mu} J_{A,\gamma}^{\mu\nu} \varepsilon_{\nu}, \quad (21)$$

it is crucial to note that this involves divergences of the axial current contributions $\hat{J}_{A,f}^{\mu}$, $\hat{J}_{A,i}^{\mu}$, and \hat{J}_{π}^{μ} according to Eqs. (10)-(12) which *do not vanish even when $q \rightarrow 0$* . The corresponding divergences of the first three and the last terms in Eq. (17), in fact, produce the complete photoproduction amplitude \mathcal{M} , plus electromagnetic contact terms arising from employing Eq. (10). Indeed, one now easily finds that

$$q_{\mu} J_{A,\gamma}^{\mu\nu} \varepsilon_{\nu} - Q_{\pi} \hat{J}_A^{\nu} \varepsilon_{\nu} = \frac{f_{\pi} \mu^2}{q^2 - \mu^2} \mathcal{M} + \bar{u}_f \mathcal{W}^{\nu} u_i \varepsilon_{\nu}, \quad (22)$$

where

$$\mathcal{W}^{\nu} = q_{\mu} W^{\mu\nu} - Q_{\pi} \hat{J}_{A,w}^{\nu}(p' - p) - G_A(q^2) N_5^{\nu}, \quad (23)$$

with

$$N_5^{\nu} = \frac{\gamma_5 \tau \Gamma_i^{\nu} + \Gamma_f^{\nu} \tau \gamma_5}{2} \quad (24)$$

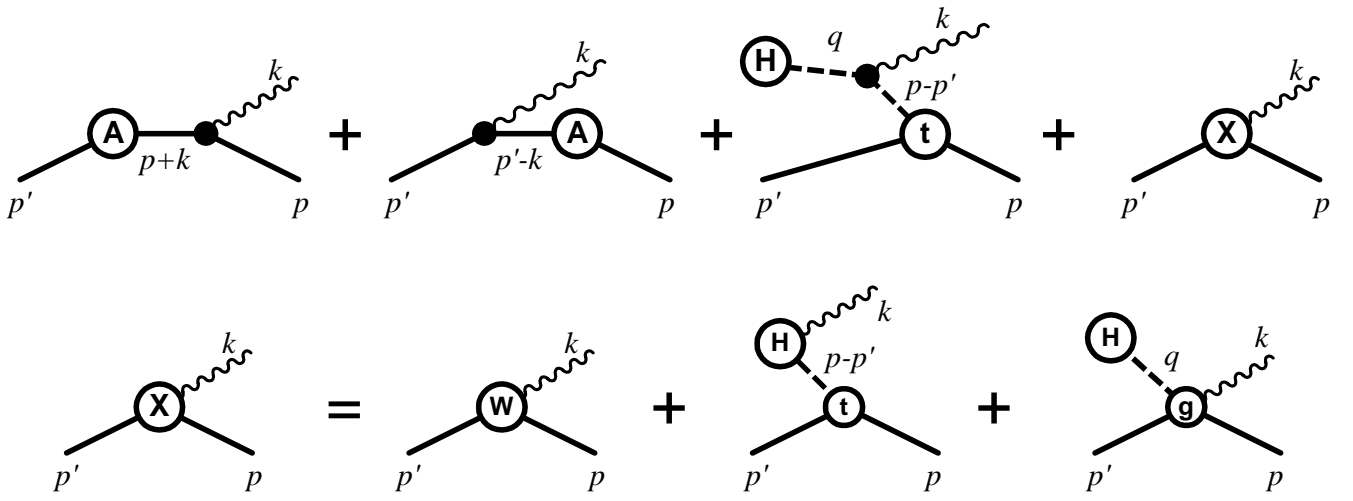


FIG. 3. Axial photoproduction current $\hat{J}_{A,\gamma}^{\mu\nu}$ of Eq. (17). The diagrams are obtained from the axial current of Fig. 1 by inserting a photon line in all possible places. In the top row, the circles denote the places where the axial current is evaluated: A corresponds to the full axial current of Fig. 1, H to its pion-pole-dominance part, and X subsumes the three terms given in the second line. The circles labeled W and H with attached photon line correspond to contact operators $W^{\mu\nu}$ and $H^{\mu\nu}$, respectively. The last diagram in that line, where the photon is interacting within the πNN vertex of the pion-pole-dominated diagram of Fig. 1, contains the interaction current M_{int}^μ of pion photoproduction (cf. Fig. 2).

being the electromagnetic contact contributions. Clearly, \mathcal{W}^ν must vanish (at least in the limit of $q \rightarrow 0$) to conform to the PCAC relation of Eq. (14).

I assume, therefore, that $q_\mu W^{\mu\nu}$ makes \mathcal{W}^ν vanish for all q , thus producing the desired result. This is consistent with the fact that in arriving at Eq. (22), the divergence $q_\mu H^{\mu\nu}$ was found to cancel $Q_\pi \hat{J}_{A,H}^\nu$ for all q .

Note that the product ansatz

$$W^{\mu\nu} \rightarrow \widehat{W}^{\mu\nu} = \frac{q^\mu}{q^2} \left[Q_\pi \hat{J}_{A,W}^\nu (p' - p) + G_A(q^2) N_5^\nu \right] \quad (25)$$

will produce $\mathcal{W}^\nu = 0$. This provides the simplest possible structure consistent with the contact nature of $W^{\mu\nu}$, i.e., that on the pion side, the only available four-momentum is q^μ and on the photon side, the only currents available are axial and electromagnetic nucleon current operators. While it is possible to write down more complicated expressions for $W^{\mu\nu}$, in view of the unknown internal structure of G_A which prevents the direct evaluation of Eq. (19), there is no basis for doing so here.

From the present considerations one may draw the following conclusions:

(a) The dependencies on the hadronic and axial form factors cancel individually via the divergences $q_\mu H^{\mu\nu}$ and $q_\mu W^{\mu\nu}$. The axial form factor G_A , in particular, appears only in \mathcal{W}^ν of Eq. (23). Therefore, since $W^{\mu\nu}$ —even though its detailed structure is not known—cannot depend on G_t , there is no additional constraint relating G_A and G_t that goes beyond the original Goldberger–Treiman relation in its generalized form (6).

(b) Assuming $\mathcal{W}^\nu = 0$, the derivation of Eq. (14) does not depend on whether q is small, i.e., it is valid for arbitrary q provided the off-shell assumptions made above about the axial currents are valid. In other words, Eq. (14) provides an alternative method of deriving the photoproduction amplitude, but it is not an independent constraint.

(c) In view of the fact that the cancellations found here require contributions from all diagrams of Fig. 3, it is evident that an incomplete or partial evaluation of these diagrams [6–10] may easily lead to erroneous conclusions.

(d) The often used approximation [8,9,11] of assuming that $q_\mu J_{A,\gamma}^{\mu\nu} \varepsilon_\nu$ vanishes for $q \rightarrow 0$ is unjustified. In fact, it is this term which provides the entire photoproduction amplitude \mathcal{M} ; the additional term $Q_\pi \hat{J}_{A,H}^\nu \varepsilon_\nu$ only serves to cancel some terms contained in $q_\mu J_{A,\gamma}^{\mu\nu} \varepsilon_\nu$ which do not contribute to \mathcal{M} . The incorrect limits are obtained if one ignores the fact that the axial currents appearing in this term must still satisfy the basic PCAC divergence constraint (3) which provides a non-vanishing result even if the momentum vanishes. Technically, the incorrect limit is obtained if one consistently drops the hadronic part $\hat{J}_{A,H}$ of Eq. (9b), and further reduces $\hat{J}_{A,W}$ of Eq. (9a) to its $\gamma_5 \gamma^\mu$ part, when evaluating Eq. (21).

(e) There is no justification in modifying the Kroll–Ruderman term by multiplying it with the axial form factor when considering virtual photons with $k^2 \neq 0$. In Ref. [6], for example, this result was obtained in what corresponds

here to but the evaluation of the first two diagrams of Fig. 3, omitting all other diagrams and $Q_\pi j_A^\nu \varepsilon_\nu$. Instead, as it was shown in Refs. [12,13], it follows from gauge-invariance requirements that the Kroll–Ruderman term must have the form

$$\hat{j}_{\text{KR}}^\mu = -\frac{\gamma_5 \gamma^\mu}{2m} G_{\text{KR}}(s, u, t) , \quad (26)$$

where G_{KR} is given as a linear combination of the s -, u -, and t -channel form factors, with appropriate charge operators. Gauge invariance does not constrain the coefficients of this linear combination. While experimental data of the momentum dependence of the Kroll–Ruderman term at threshold may help fix these coefficients, they clearly do not have any bearing on the properties of the axial form factor, as was shown here.

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